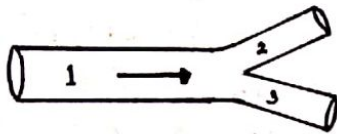


X	Y	Z
7	1	7

1. Aliran air mengalir dalam pipa 1 dengan jari-jari 7,87 cm, kemudian bercabang menjadi pipa 2 & 3 dengan diameter $d_2 = 0,31 d_1$ dan $d_3 = 0,5 d_1$. Kecepatan pipa 2 $0,47 V_1$. Hitung debit aliran bila V_{max} semua pipa tidak diperbolehkan melebihi $5,17 \text{ m/s}$.



- $d_1 = 7,87 \times 2 = 15,74 \text{ cm} = 0,1574 \text{ m}$
- $d_2 = 0,31 \times 0,1574 = 0,0488 \text{ m}$
- $d_3 = 0,5 \times 0,1574 = 0,0787 \text{ m}$
- $V_2 = 0,47 V_1$
- $V_{max} = 5,17 \text{ m/s}$

- $Q_1 = Q_2 + Q_3$

$$A_1 \cdot V_1 = A_2 \cdot V_2 + A_3 \cdot V_3$$

$$\pi \cdot \frac{d_1^2}{4} \cdot V_1 = \pi \cdot \frac{d_2^2}{4} \cdot V_2 + \pi \cdot \frac{d_3^2}{4} \cdot V_3$$

$$d_1^2 \cdot V_1 = d_2^2 \cdot V_2 + d_3^2 \cdot V_3$$

$$(0,1574)^2 V_1 = (0,0488)^2 \cdot 0,47 V_1 + (0,0787)^2 \cdot V_3$$

$$0,02477476 V_1 = 0,001119277 V_1 + 0,00619369 V_3$$

$$0,023655483 V_1 = 0,00619369 V_3$$

$$V_3 = 3,82 V_1$$

- $Q_1 = A_1 \cdot V_1$

$$= 3,14 \times \frac{(0,1574)^2}{4} \times 0,977$$

$$= 0,019 \text{ m}^3/\text{s}$$

- $Q_2 = A_2 \cdot V_2$

$$= 3,14 \times \frac{(0,0488)^2}{4} \times 0,459$$

$$= 0,001 \text{ m}^3/\text{s}$$

- $Q_3 = A_3 \cdot V_3$

$$= 3,14 \times \frac{(0,0787)^2}{4} \times 3,732$$

$$= 0,018 \text{ m}^3/\text{s}$$

- Cek

$$Q_1 = Q_2 + Q_3$$

$$0,019 = 0,001 + 0,018$$

$$0,019 = 0,019 \quad (\text{OK!})$$

- $V_{max} = V_1 + V_2 + V_3$

$$5,17 = V_1 + 0,47 V_1 + 3,82 V_1$$

$$V_1 = \frac{5,17}{5,29} = 0,977 \text{ m/s}$$

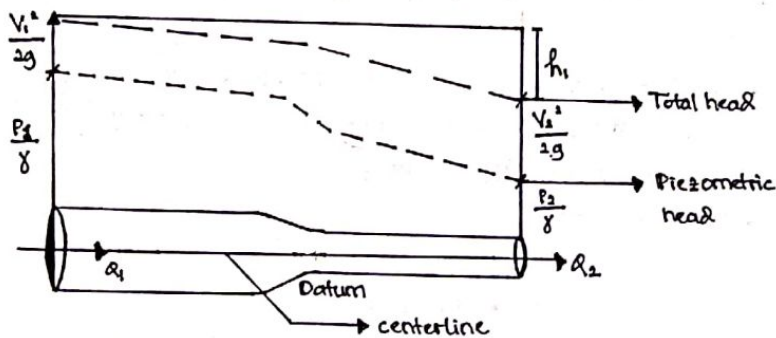
$$V_2 = 0,47 \times 0,977$$

$$= 0,459 \text{ m/s}$$

$$V_3 = 3,82 \times 0,977$$

$$= 3,732 \text{ m/s}$$

2. Di sebuah Desa di Yogyakarta terdapat pipa yang mengalirkan air secara horizontal dengan panjang 179 m. Pada daerah hulu, pipa tersebut memiliki bagian diameter 24,71 cm, di bagian hilir, diameter mengecil menjadi setengahnya. Pada daerah hulu dan hilir, tekanan masing-masing pipa memiliki perbedaan 2,1 kgf/cm². Estimasi debit aliran yang terjadi.



- panjang pipa = 179 m
- $D_1 = 24.71 \text{ cm} = 0.2471 \text{ m}$
- $D_2 = \frac{1}{2} D_1 = \frac{1}{2} 0.2471 = 0.12355 \text{ m}$
- $\gamma_{\text{air}} = 1000 \text{ kg/m}^3$
- $\Delta P = P_1 - P_2 = 2.1 \text{ kgf/cm}^2 = 21 \text{ ton/m}^2$
- $Q = ?$

• Tinggi tekanan₁ = $\frac{P_1}{\gamma} = \frac{P_1}{1000}$

Tinggi tekanan₂ = $\frac{P_2}{\gamma} = \frac{P_1 - 21}{1000}$

• $V_1 = 0.25 V_2$
 $= 0.25 \times 0.67$
 $= 0.1675 \text{ m/s}$

• $Q_1 = Q_2$

$A_1 \cdot V_1 = A_2 \cdot V_2$

$\pi \cdot \frac{(0.2471)^2}{4} V_1 = \pi \cdot \frac{(0.12355)^2}{4} V_2$

$V_1 = \frac{0.0153}{0.0610} V_2$

$V_1 = 0.25 V_2$

• $Q_1 = A_1 \cdot V_1$
 $= 3.14 \times \frac{(0.2471)^2}{4} \times 0.1675$
 $= 0.008 \text{ m}^3/\text{s}$

• $Q_2 = A_2 \cdot V_2$
 $= 3.14 \times \frac{(0.12355)^2}{4} \times V_2$
 $= 0.008 \text{ m}^3/\text{s}$

• $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$
 $\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$
 $\frac{21}{1000} = \frac{V_2^2}{2 \cdot 10} - \frac{(0.25 V_2)^2}{2 \cdot 10}$

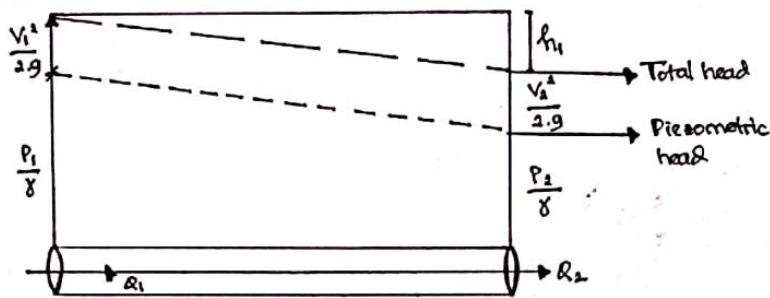
$0.021 = \frac{V_2^2 - 0.0625 V_2^2}{20}$

$0.42 = 0.9375 V_2^2$

$V_2 = \sqrt{\frac{0.42}{0.9375}}$
 $= 0.67 \text{ m/s}$

• $Q_1 = Q_2$ (ok!)

3. Dengan kasus yang sama dengan nomor 2. Pertanyaannya, di hilir tidak ada perubahan diameter pipa. Sehingga, diameternya sama. Bila koefisien gesek aliran yang terjadi adalah $f = 0,0377$, tentukan debit aliran.



$$\bullet d_1 = d_2 = 0,2471$$

$$\bullet f = 0,0377$$

$$\bullet f = 0,316$$

$$Re^{0,25}$$

$$Re^{0,25} = 0,316$$

$$0,0377$$

$$Re^{0,25} = 8,382$$

$$Re = 4936,17$$

(aliran turbulen)

$$\bullet \frac{1}{\sqrt{f}} = -2 \log \left(\frac{K}{3,7 \cdot d} + \frac{2,51}{Re \sqrt{f}} \right)$$

$$\frac{1}{\sqrt{0,0377}} = -2 \log \left(\frac{K}{3,7 \times 0,2471} + \frac{2,51}{4936,17 \sqrt{0,0377}} \right)$$

$$5,155 = -2 \log \left(\frac{K}{0,914} + \frac{2,51}{957,6} \right)$$

dimisalkan x

$$5,155 = -2 \log x$$

$$x = 10^{(5,155/2)}$$

$$x = 378,007$$

$$\left(\frac{K}{0,914} + \frac{2,51}{957,6} \right) = 378,007$$

$$\frac{K}{0,914} = 378,007 - 0,0026$$

$$K = 378,004 \times 0,914$$

$$= 345,5$$

$$\bullet K = f \cdot \frac{L}{d}$$

$$= \frac{0,0377 \cdot 179}{0,2471}$$

$$= 27,31$$

$$\bullet Q_1 = Q_2$$

$$A_1 \cdot V_1 = A_2 \cdot V_2$$

$$\pi \cdot \frac{d_1^2}{4} \cdot V_1 = \pi \cdot \frac{d_2^2}{4} \cdot V_2$$

$$V_2 = \left(\frac{d_1}{d_2} \right)^2 V_1$$

$$\bullet K = \frac{V_2^2}{2g} \cdot \frac{L}{d} \cdot f$$

$$27,31 = \frac{V_2^2}{2 \cdot 10} \cdot \frac{179}{0,2471} \cdot 0,0377$$

$$V_2^2 = \frac{27,31 \cdot 20}{27,31}$$

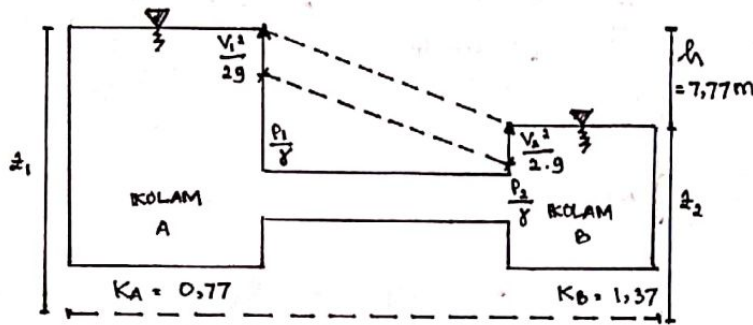
$$V_2 = \sqrt{20} = 4,47 \text{ m/s}$$

$$\bullet Q_1 = Q_2 = A_2 \cdot V_2$$

$$= 3,14 \cdot \frac{(0,2471)^2 \cdot 4,47}{4}$$

$$= 0,214 \text{ m}^3/\text{s}$$

4. Dengan kasus yang sama dengan nomor 3. Bedanya, pipa mengalir melalui kolam A dan B dengan pipa tersebut. Elevasi perbedaan kolam di hulu dan hilir adalah 7,77 m. Bila koefisien kehilangan teraga akibat perubahan penampang pada sambungan antara pipa dan kolam A dan B adalah $K_A = 0,77$, $K_B = 1,37$. Estimasi debit aliran.



$$1. \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \Sigma h + \Sigma h_f$$

$$h + h_f = \left(\frac{z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g}}{\gamma} \right) - \left(\frac{z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}}{\gamma} \right)$$

$$= \left(\frac{z_1 + \frac{P_1}{\gamma}}{\gamma} \right) - \left(\frac{z_2 + \frac{P_2}{\gamma}}{\gamma} \right)$$

$$h_{db} + h_{fi} = H$$

$$K_A \cdot \frac{V_1^2}{2g} + K_B \cdot \frac{V_2^2}{2g} + f_1 \cdot \frac{L_1}{d_1} \cdot \frac{V_1^2}{2g} = H \quad \dots \text{ (Persamaan I)}$$

$$2. Q_1 = Q_2$$

$$A_1 \cdot V_1 = A_2 \cdot V_2$$

$$V_2 = \frac{A_1}{A_2} \cdot V_1$$

$$V_2 = \frac{\frac{1}{4} \pi \cdot d_1^2 \cdot V_1}{\frac{1}{4} \pi \cdot d_2^2}$$

$$V_2 = \frac{d_1^2}{d_2^2} V_1 \quad \dots \text{ (persamaan II)}$$

$$3. \text{ Substitusi (II) ke (I)}$$

$$\left(K_A + K_B \left(\frac{d_1}{d_2} \right)^4 + f_1 \frac{L_1}{d_1} \right) \frac{V_1^2}{2g} = 7,77$$

$$\left(0,77 + 1,37(1) + 0,0577 \frac{179}{0,2471} \right) \frac{V_1^2}{20} = 7,77$$

$$(0,77 + 1,37 + 27,31) V_1^2 = 155,4$$

$$V_1^2 = 5,276$$

$$V_1 = 2,3 \text{ m/s}$$

$$4. \text{ Jika } f_2 \text{ diperhitungkan}$$

$$\frac{K_A}{2g} V_1^2 + K_B \left(\frac{d_1}{d_2} \right)^4 \frac{V_1^2}{2g} + f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{d_2} \frac{V_2^2}{2g} = H$$

$$\left(K_A + K_B \left(\frac{d_1}{d_2} \right)^4 + f_1 \frac{L_1}{d_1} + f_2 \frac{L_2}{d_2} \left(\frac{d_1}{d_2} \right)^4 \right) \frac{V_1^2}{2g} = H$$

$$(0,77 + 1,37 + 27,31 + 27,31) V_1^2 = 7,77 \times 20$$

$$V_1^2 = 2,7378$$

$$V_1 = 1,65 \text{ m/s}$$

$$V_2 = \left(\frac{d_1}{d_2} \right)^2 V_1$$

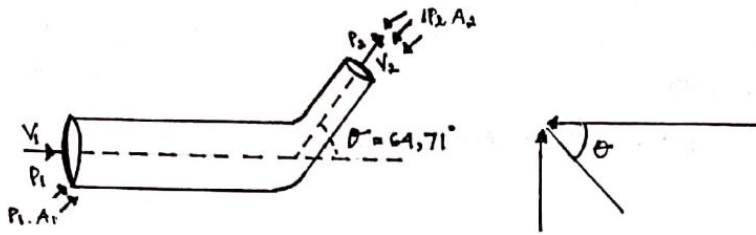
$$V_2 = V_1$$

$$5. Q = A \cdot V$$

$$= 3,14 \times \frac{(0,2471)^2 \cdot 1,65}{4}$$

$$= 0,079 \text{ m}^3/\text{s}$$

7. Dengan kasus yang sama dengan nomor 2, bejana airnya mengalir pada pipa yang membesar dari diameter besar ke diameter kecil dengan sudut belokan 64,71°. Panjang pipa diabaikan. Estimasi kecepatan dengan kasus tersebut, kemudian presisi gaya untuk menahan pipa bila tekanan yang terjadi mengalami kenaikan 14,17% dari tekanan studi kasus awal.



$$P_1 \cdot A_1 - P_2 \cdot A_2 \cos \theta - R_x = \rho Q (V_2 \cos \theta - V_1)$$

arah x $\rightarrow R_x = P_1 \cdot A_1 - P_2 \cdot A_2 \cos \theta - \rho Q (V_2 \cos \theta - V_1)$

arah y $\rightarrow R_y = W + P_2 \cdot A_2 \sin \theta - \rho Q V_2 \sin \theta$

$$R_y = W + P_2 \cdot A_2 \sin \theta + \rho Q V_2 \sin \theta$$

$$Q_1 = Q_2 \quad \left| \quad V_2 = \frac{0,0479 \cdot 0,1675}{0,012} \right.$$

$$A_1 \cdot V_1 = A_2 \cdot V_2 \quad \left| \quad = 0,668 \text{ m/s} \right.$$

$$V_2 = \frac{A_1 \cdot V_1}{A_2}$$

$$Q_1 = A_1 \cdot V_1$$

$$= 0,0479 \cdot 0,1675$$

$$= 0,008$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2 \cdot g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2 \cdot g}$$

$$\frac{P_1}{1000} + \frac{(0,1675)^2}{20} = \frac{P_2}{1000} + \frac{(0,668)^2}{20}$$

$$\frac{P_1}{1000} - \frac{P_2}{1000} = \frac{0,668^2 - 0,1675^2}{20}$$

$$P_1 - P_2 = \frac{0,5005}{20} \cdot 1000$$

$$P_1 - P_2 = 25,025 \text{ N/m}^2$$

- Jika P_1 adalah awal pada no. 2, maka :

$$P_1 = 8,375 \text{ N/m}^2$$

$$P_2 = 25,025 - 8,375 = 16,65 \text{ N/m}^2$$

Terjadi kenaikan 14,17%

$$P_1 = 8,375 + 8,375 \times 14,17\%$$

$$= 8,375 + 1,19 = 9,56 \text{ N/m}^2$$

$$P_2 = 8,375 + 8,375 \times 14,17\%$$

$$= 9,56 \text{ N/m}^2$$

! catatan

$$P_1 = P_1 + 14,17 P_1$$

$$P_2 = P_2 + 14,17$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2 \cdot g} = 0$$

$$\frac{P_1}{1000} + \frac{0,1675^2}{20} = 0$$

$$\frac{P_1}{1000} + \frac{0,1675^2}{20} = 0$$

$$P_1 = 8,375 \text{ N/m}^2$$

$$\begin{aligned}
 \rightarrow R_x &= P_1 \cdot A_1 - P_2 \cdot A_2 \cos \theta - pR (V_2 \cos \theta - V_1) \\
 &= 9,96 \cdot 0,0179 - 9,96 \cdot 0,012 \cdot \cos 64,71 = \frac{1000}{10} \times 0,008 (0,668 \cos 64,71 - 0,1675) \\
 &= 0,177 - 0,036 - 0,026 \\
 &= 0,115 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow R_z &= \boxed{W} + P_2 \cdot A_2 \cdot \sin \theta + pR V_2 \sin \theta \\
 &= 9,96 \cdot 0,012 \cdot \sin 64,71 + \frac{1000}{10} \times 0,008 (0,668 \sin 64,71) \\
 &= 0,1135 + 0,5092 \\
 &= 0,623 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow R &= \sqrt{R_x^2 + R_z^2} \\
 &= \sqrt{0,115^2 + 0,623^2} \\
 &= \sqrt{0,132 + 0,388} \\
 &= \sqrt{0,52} \\
 &= 0,721 \text{ N}
 \end{aligned}$$